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On Modeling a Rolling Wheel as a Two- or Three-dimensional Process

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1 INTRODUCTION

The process of a wheel rolling over material in the presence of wheel force large enough to induce permanent deformation has often been treated as two-dimensional (2D), with the dimension associated with the width of the wheel removed from consideration. In such theoretical works (e.g., [1-3]), plane strain and rigid-perfect plasticity were typically assumed to enable analysis based on the method of characteristics. Three-dimensional (3D) analyses using the finite element method [4-6] or an approximate analytic technique derived from solutions for limit loads in punch indentation [5,6] have only recently been proposed. While the assumption of plane strain may be valid for a wide wheel (i.e., a cylinder), one finds that deformation out of the diametral plane prevails when the wheel is narrow. Typical wheels on machines designed for minimal rolling resistance or maximal tractive force have width-to-radius ratios between 0.2 and 0.6 and therefore cannot be considered wide.

Based on numerical simulation using the finite element method, this work provides a detailed comparison of the fundamental differences arising between 2D and 3D analyses of a wheel rolling quasi-statically on an elastoplastic material. The case of a towed wheel is considered, meaning that no torque is applied at the wheel's axis of rotation, and a rigid wheel is assumed.

2 NUMERICAL SIMULATION

Beginning with indentation, a rolling process comprising an initial transient regime and a steady-rolling regime, provided one could exist for the parameters chosen, was simulated as described in [5-6] using the finite element code ABAQUS/Explicit. The soil was modeled as isotropic and elastic-perfectly plastic with linear elasticity (Young's modulus E; Poisson's ratio ν) and plasticity governed by the von Mises yield condition (shear strength $k = \sigma_o/\sqrt{3}$, where σ_o is uniaxial yield strength) with associated plastic flow. Isotropic Coulomb friction with coefficient of friction μ was prescribed as the contact interaction between the soil and torque-free wheel (diameter d; width b). Simulation consisted of applying vertical force W to the wheel while it was placed on a region of relatively strong material in which little penetration occurred and then prescribing horizontal wheel velocity such that the wheel rolled into a region of weak material. Simulation proceeded until either a steady state was reached or unstable wheel penetration occurred. Steady state was identified with the condition that wheel penetration s, horizontal force H, and wheel rotational velocity ω did not vary with respect to horizontal wheel displacement u. The simulations also provided the geometry of the free surface in steady state. In previous 2D analyses [1-3], the shape of the free surface needed to be postulated *a priori*.

3 RESULTS AND DISCUSSION

As shown in Fig. 1, the simulations reveal that, in steady state, the dependence of normalized horizontal force ($\overline{H} = H/kbr$) on normalized vertical force ($\overline{W} = W/kbr$) is qualitatively similar whether 2D or 3D analysis is utilized, whereas wheel penetration and material deformation can be drastically different. When the process is modeled in 3D, simulations show that normalized wheel penetration ($\overline{s} = s/r$) in steady state depends monotonically on $\overline{W}$. Based on 2D simulations, however, one observes that $\overline{s}$ is roughly constant over some interval of $\overline{W}$ (approximately $0.6 < \overline{W} < 2.4$ for $\overline{E} = E/k = 690$, $\nu = 0.3$, and $\mu = 0.5$). For $\overline{W}$ below this interval, deformation is largely elastic. For $\overline{W}$ above this interval, wheel force is too large for a steady configuration to exist, and wheel penetration becomes unstable.

The key difference arising between 2D and 3D simulations involving standard wheels ($0.2 < b/r < 0.6$) is the tendency for material to accumulate in front of the wheel (2D) rather than flow around (3D) when material plasticity is dominating. A basic similarity between 2D and 3D is that an increase in wheel force $\overline{W}$ is sustained by the material principally through an increase in contact area, as opposed to an increase in contact stresses acting over that area. Thus for processes modeled in 2D, an increase in $\overline{W}$ causes an increase in accumulation and a corresponding change in contact area, without significant change in $\overline{s}$. This is distinctly different from the 3D process, where accumulation is secondary and contact area evolves almost exclusively through $\overline{s}$. Similar observations were noted for indentation [7], in which accumulation plays no role and the force-penetration curve is relatively insensitive to whether 2D or 3D analysis is considered.

4 REFERENCES

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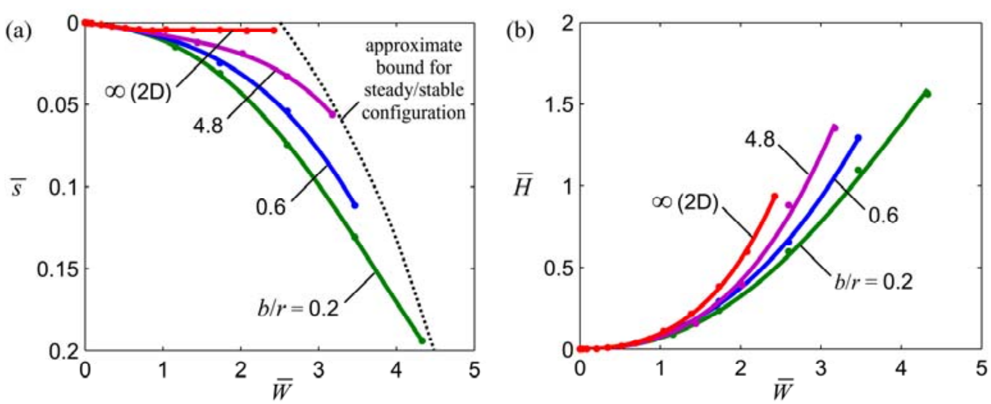

Figure 1. Normalized steady-state penetration (a) and horizontal force (b) against normalized vertical force for towed wheel with varying aspect ratio ($\overline{E} = 690$, $\nu = 0.3$, and $\mu = 0.5$)