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Limit Loads for Pipelines and Cylinders Partially Embedded in Frictional Materials

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ABSTRACT: Accurately assessing the forces and reactions on cylindrical objects partially embedded in soils is fundamental in pipeline engineering as well as the analysis and design of off-road vehicles. With a view towards understanding the limiting forces acting over the contact area between the cylinder and the soil, this paper compares two different theoretical approaches for predicting ultimate loads under combined vertical and horizontal loading. The first is an approximate method based on idealizing the contact interface as a flat shallow strip footing, and the second is based on finite element limit analysis (FELA). Focus is on purely frictional material and plane strain conditions, assuming a rigid cylinder and perfectly plastic material obeying the Mohr-Coulomb yield condition with associated plastic flow. The approximate method based on an analogy to an equivalent shallow foundation (with no overburden) predicts results that are reasonably close to those from FELA but only when small embedment is considered. Moreover, the discrepancy between the two approaches for the case of free cylinder rotation is attributed to the existence of a contact moment over the soil-cylinder contact interface. Practical implications for pipeline engineering and off-road vehicle engineering are discussed.

1. INTRODUCTION

The evaluation of ultimate contact force between partially embedded (on-bottom) cylindrical structures and soil is a fundamental problem for applications in various branches of engineering, from assessing the stability of offshore pipelines (Small, 1971; Gao et al. 2015) to determining the forces on wheeled vehicles operating on soils (Collins, 1972; Hambleton and Stanier, 2017). For instance, soil resistance against lateral buckling of pipelines is critical for the design of oil pipelines which can buckle due to thermal expansion, mechanical actions or geo-hazards. Accurately predicting resistance not only creates economic benefits but also prevents potentially significant impacts on the environment resulting from oil leakage (Oliveira et al. 2010).

A literature review shows that various approaches have been proposed to assess the ultimate (limit) loads on partially embedded cylinders. Based on limit analysis, Murff et al. (1989) derived upper bound and lower bound solutions for the ultimate vertical load acting on a pipeline embedded in perfectly plastic cohesive material. Using the upper bound approach of limit analysis, Randolph and Houlsby (1984) and Martin and Randolph (2006) studied the capacity of a pile against lateral loading in perfectly plastic cohesive soil, which can be used to analyze very deeply embedded pipelines whose bearing capacity is independent of the embedment depth and the loading direction. By idealizing soil as a perfectly plastic cohesive soil, Randolph and White (2008) proposed an upper bound solution to evaluate the bearing capacity of shallowly embedded pipelines subjected to combined vertical and horizontal loading. Their theoretical predictions were presented as V-H failure envelopes, where the variables V and H refer to the vertical and horizontal components of the ultimate load, respectively. As a verification of Randolph and White's upper bound solution, Merifield et al. (2008) performed a finite element (FE) analysis of the same problem. Using finite element limit analysis (FELA), Martin and White (2012) studied the limit loads for both partially and fully embedded offshore pipelines under general loading. Again, the soil in this work is assumed to be purely cohesive. It is noted that most of theoretical works regarding the ultimate loads on pipelines has focused on cohesive soils, or the case of undrained loading. Studies on the limit loads for partially embedded cylinders in frictional material (e.g., sand) are comparatively scarce. However, this area is of equal importance for engineering practice. Examples of practical importance include pipelines sitting on sand/silt and subjected to slow (undrained) loading, and the performance of vehicles which are designed to operate on the beach. Gao et al. (2015) proposed a slip-line solution to predict the vertical bearing capacity of pipelines partially embedded in drained soils. Based on experimentally obtained V-H failure envelopes, Zhang et al. (2012) proposed a bounding surface model to compute the force-displacement relation of a shallowly embedded pipeline in calcareous sand subjected to general loading conditions.

In engineering practice, the assessment of the limit load for a partially embedded cylinder or pipeline is sometimes simplified by evaluating the bearing capacity of an equivalent shallow strip foundation (Small, 1971; Brown, 2001; O'Rourke and Liu 2012). Hambleton and Stanier (2017) applied the same strategy to predict the forces acting on the wheels of off-road vehicles, by again considering an analogous strip foundation at the interface between wheel and soil. In this way, the forces developing on the wheel are related to soil properties through existing bearing capacity theories. This approach based on the shallow foundation analogy is appealing for its simplicity compared with the aforementioned analytical or numerical approaches, but more importantly this simplification enables direct application of a body of knowledge regarding foundation bearing capacity for the purposes of predicting the limit loads on cylindrical objects.

O'Rourke and Liu (2002) showed that the equivalent shallow foundation model can provide reasonably accurate prediction of vertical bearing capacity of pipelines as compared with more sophisticated methods. Kouretzis et al. (2015) concluded that this is valid only for deeply embedded pipes and loose-to-medium dense sand. Additional investigation is needed to assess whether this simplification is satisfactory when used to predict the limit loads for cylinders subjected to general loading (i.e., combined horizontal and vertical loading). This is the key question that this work attempts to answer. For this purpose, finite element limit analysis (FELA) was utilized to compute the limit loads for both a partially embedded cylinder and the corresponding equivalent shallow foundation, and comparisons are presented in the form of V-H failure envelopes. The analyses completed for this work were restricted to plane strain, a rigid cylinder and perfectly plastic cohesionless soil obeying the Mohr-Coulomb yield condition with associated plastic flow. Deviating from previous studies in which only translational movement was considered, this work also studies the impact of the cylinder's rotation on the limit load, which is particularly vital for applications pertaining to off-road vehicles.

2. SHALLOW FOUNDATION ANALOGY

Figure 1 illustrates the shallow foundation analogy used to assess the limit load for a partially embedded cylinder. As a hypothesis, the evaluation of the ultimate force acting on the cylinder, denoted by F and depicted in Fig. 1(a), is equivalent to assessing the ultimate force acting on a strip foundation (no overburden) located at the interface between soil and cylinder. In additional to the vertical and horizontal components of loading, V and H, a moment M may also act at the interface, as shown in Fig.1(b). The width of the strip foundation is related to both the diameter D and embedment depth z of the cylinder:

$$B = \sqrt{\left(\frac{D}{2}\right)^2 - \left(\frac{D}{2} - z\right)^2} \tag{1}$$

At the limit load F, the soil supporting the cylinder exhibits plastic flow in the form of large and permanent deformation. At the point of first yielding, before significant geometric changes have occurred, it is reasonable to assume that the ultimate force developed on the soil-cylinder interface, which balances the load applied at the center of the cylinder, corresponds to the combination of V, H and M that causes yielding of soil beneath the equivalent strip foundation (i.e., bearing capacity problem). Considering equilibrium of the cylinder, F can be related to V, H, and M as follows:

$$F \sin\alpha = V \tag{2}$$

$$F \cos\alpha = H \tag{3}$$

$$F \cos\alpha \left(\frac{D}{2} - z\right) = M \tag{4}$$

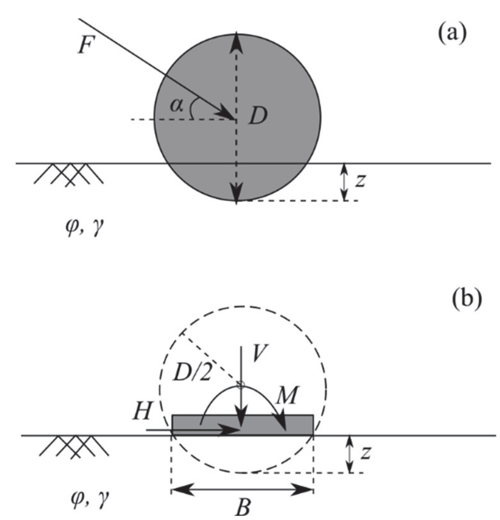

Fig. 1. Schematic showing: (a) inclined ultimate force on a partially embedded cylinder; (b) ultimate forces on an analogous shallow foundation

It is noted that the vertical and horizontal force components, V and H, do not depend on the point of application of the force F and thus refer generally to the vertical and horizontal loading on the pipe, as well as the vertical and horizontal force acting over the interface. However, the contact moment M, depends on where the force F is applied. In this paper, and Eq. (4) in particular, the force F is taken to act at the center of the cylinder.

3. FINITE ELEMENT LIMIT ANALYSIS

By integrating concepts employed in the finite element method (namely, spatial discretization and field interpolation by shape functions) with the upper and lower bound theorems of limit analysis, finite element limit analysis (FELA) provides an efficient means of evaluating the collapse loads acting on perfectly plastic materials (Lyamin and Sloan, 2002a; Lyamin and Sloan, 2002b; Krabbenhøft, Lyamin and Sloan, 2008). Depending on the assumed yield surface, the assessment of the limit load is formulated as a linear or non-linear optimization problem. The upper bound solution is obtained by optimizing a kinetically admissible velocity field, whereas a lower bound on the limit load is obtained by optimizing a stress field that satisfies equilibrium and boundary conditions (Chen, 1975). In this work, the limit loads on a cylinder and strip foundation were computed using the commercial FELA software OptumG2 (Krabbenhøft and Lyamin, 2014).

3.1. Benchmark problem: Strip foundation on cohesionless soil

To assess the simulation requirements for accurately determining the limit loads, the bearing capacity problem of a strip foundation subjected to vertical loading was first analyzed. The friction angle ϕ is assumed to be $\phi = 40°$, and limit load is normalized by the width of the foundation B and unit weight of the soil γ. Two different values for the friction angle at the soil-structure interface, or so-called wall friction denoted by ϕ_w, are assumed: $\phi_w = 0$ (smooth interface) and $\phi_w = \phi$ (rough interface). The simulations were completed using a rectangular region of soil with a width of 35B and depth of 15B, both sufficiently large that boundary effects are demonstrably negligible.

Figure 2 presents the computed upper and lower bounds for the vertical bearing capacity ($V = V_{max}$, $H = 0$). The exact solutions obtained by Martin (2005) using the method of characteristics are included for comparison. As the number of elements increases, both the calculated upper and lower bounds approach the exact solution, indicating the collapse loads are predicted to within a few percent of the true solution. The results shown in the

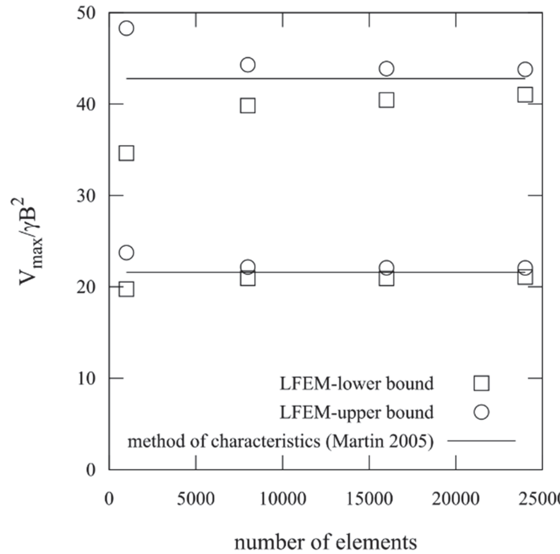

Fig. 2. Comparison of the ultimate bearing capacity of a strip foundation on cohesionless soil computed by FELA with the exact solution obtained using the method of characteristics (Martin, 2005)

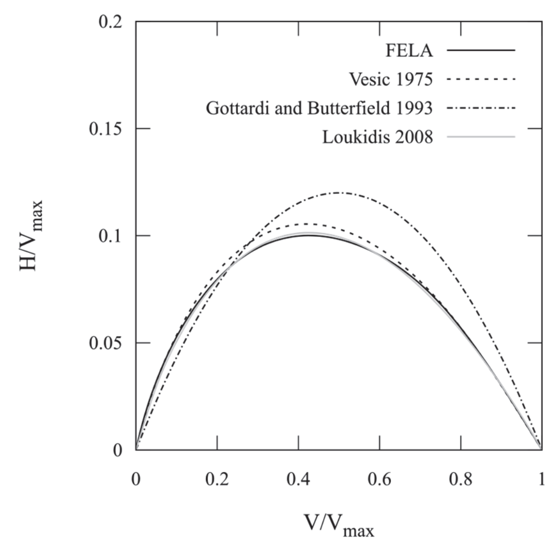

Fig. 3. Comparison of normalized V-H failure envelopes predicted using FELA with various existing relationships

figure are the outcome of trialing various meshing strategies, using adaptive mesh refinement in each case.

Ultimately, the selected mesh included roughly 24,000 elements in total, obtained after 4 iterations of adaptive mesh refinement with the initial number of elements set to 1,000. For the remainder of the paper, the mean value of the upper and lower bounds is regarded as the computed limit load.

The interaction between soil and structure under general loading is of particular interest to this work. Figure 3 shows the normalized V-H failure envelope computed by FELA using the same simulation procedure described above except with an inclined rather than vertical load.

The computed average of the upper and lower bounds is shown along with results from other methods: an existing analytical solution (Vesic, 1975), experiments (Gottardi and Butterfield, 1993) and the displacement-based finite element method (Loukidis, 2008). As seen, FELA predicts a normalized V-H envelope that lies close to the existing theoretical solutions.

3.2. *Limit loads on partially embedded cylinder*

After benchmarking the numerical solution with the strip foundation problem, similar analyses were conducted to compute limit loads acting on planar cylinder embedded at various depths. A friction angle of $\phi = 40°$ was again assumed, and three different embedment depths were considered: z/D = 0.1, 0.3, and 0.5, where z is the embedment depth as shown in Figure 1(a). The computed limit loads are presented in the form of normalized V-H failure envelopes (Figure 4). For each embedment depth, smooth and rough contact conditions at the soil-cylinder interface are considered. Furthermore, two different scenarios are included to account for the rotational degree of freedom in the problem. In one case, the cylinder is fixed again rotation, such that it can only translate vertically and horizontally. This is achieved by fixing the rotation at the center of the cylinder, where the limit load F is applied. In the second case, the cylinder is free to rotate. Generally, zero rotation represents a long pipe, where torsion should be negligible, or a locked (braked) wheel in the mobility problem.

From Figure 4, one can observe that a cylinder located at deeper embedment can mobilize higher soil resistances along both the vertical and horizontal directions. In other words, it has a higher capacity to resist external loads. When embedded at the same depth, the cylinder with a rough interface can sustain significantly higher vertical and horizontal loads than that with a smooth interface. Regarding the impacts of rotation, one can see that the cylinder that can rotate freely generally has a lower capacity to resist external loads. Nevertheless, this reduction in capacity is only significant when the cylinder is shallowly embedded (i.e., z/D < 0.3) and the interface is rough.

4. COMPARISON BETWEEN LIMIT LOADS FOR CYLINDER AND EQUIVALENT FOUNDATION

In this work, the width of the strip foundation equivalent to a planar cylinder with diameter of D and embedment of z is calculated from Eq. (1). Figure 5 compares the limit loads acting on the cylinder and the corresponding equivalent strip foundation. In this comparison, only the rough soil-structure interface was considered, recognizing that a smooth interface yields no capacity in the horizontal direction for a strip foundation. It is worth mentioning that when analyzing the equivalent strip foundation, moments acting on the foundation are not considered, i.e., both vertical and horizontal loads are applied at the centroid of the foundation. The same simplification was also used by Hambleton and Stanier (2017) and Collins (1972) to predict the forces occurring as the wheels of an off-road vehicle interact with soil.

From Figure 5, it can be noted that the equivalent shallow foundation model reasonably predicts the limit loads on shallowly embedded cylinders (i.e., $z/D \leq 0.3$ in this work), with the maximum discrepancy on the vertical and horizontal limit forces being between 17% and 30%, respectively. In particular, the equivalent foundation model predicts very well the limit loads of the cylinder whose embedment is very shallow (z/D = 0.1) and whose rotation is constrained. On the other hand, for the deeply embedded cylinder (i.e., z/D > 0.3), both the vertical and horizontal limit loads are significantly underestimated (30% and 50%, respectively for z/D = 0.5) by the equivalent shallow foundation model.

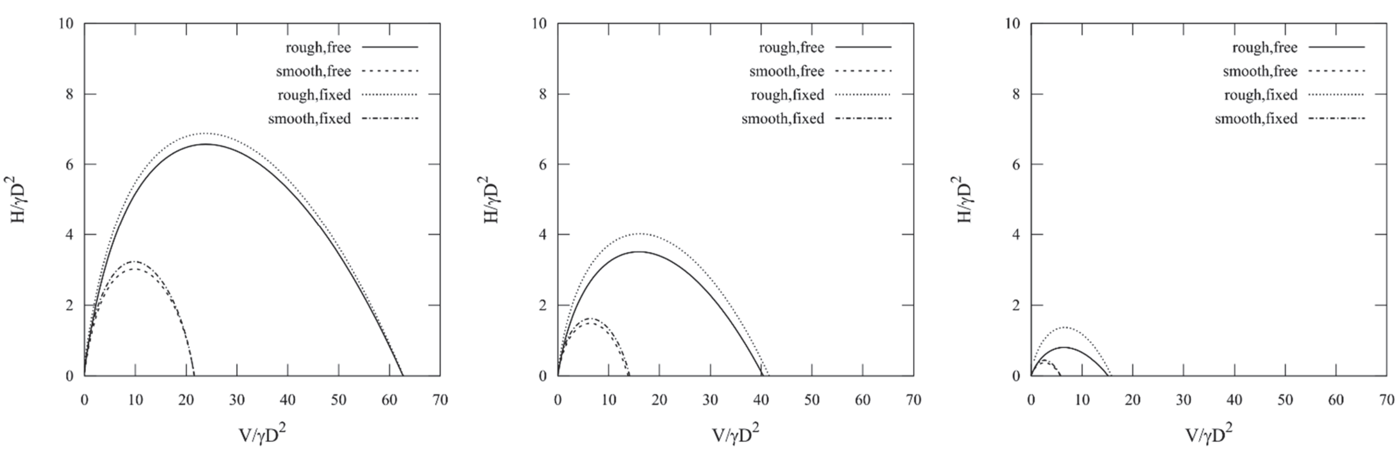

Fig. 4. Computed limit loads on pipelines with different embedded depths: (a) z/D = 0.5; (b) z/D = 0.3; (c) z/D = 0.1

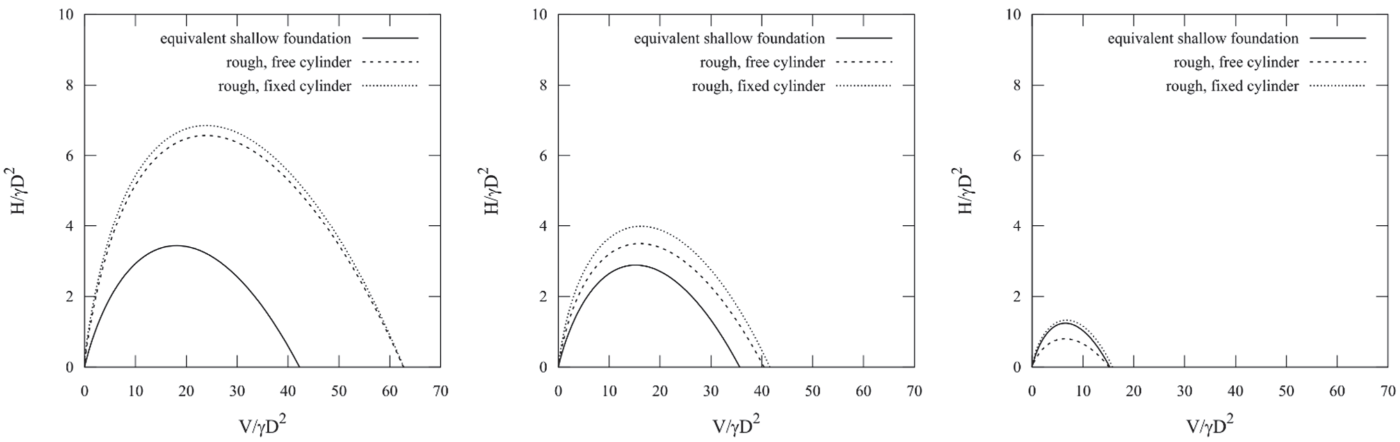

Fig. 5. Comparison of limit loads for a cylinder with those for an equivalent strip foundation: (a) z/D = 0.5; (b) z/D = 0.3; (c) z/D = 0.1

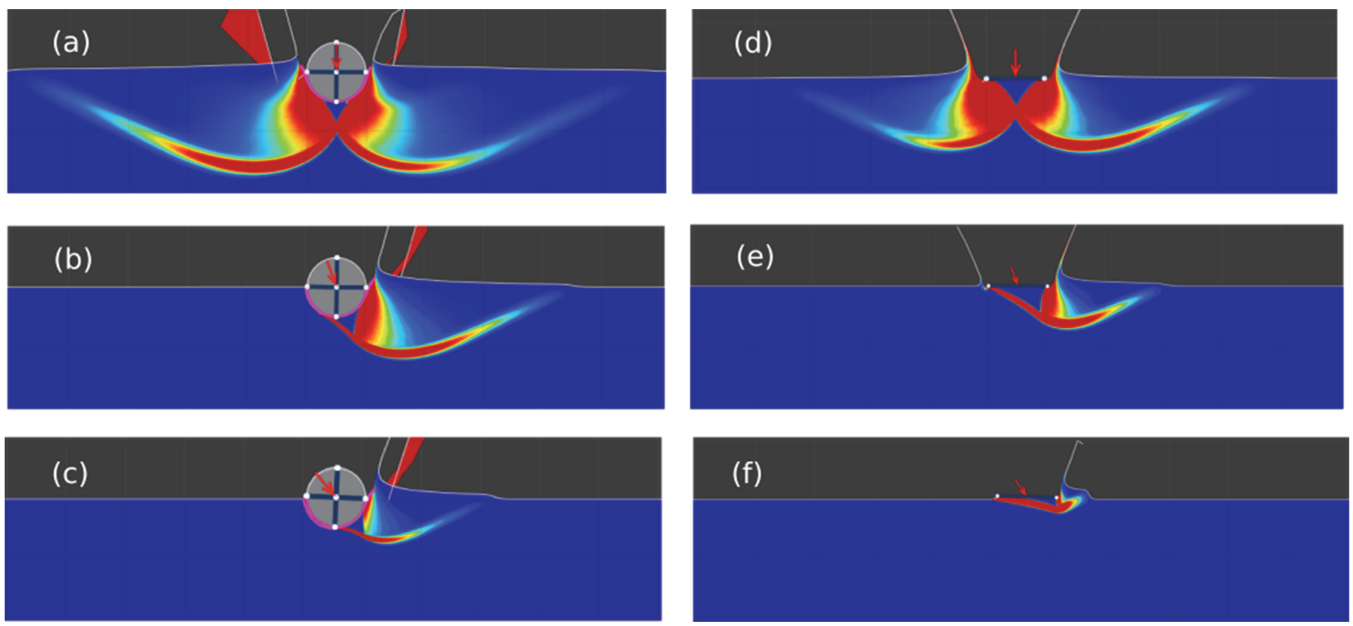

Fig. 6. Intensity of plastic strain rate in upper bound analysis indicating failure mechanism for soils beneath a cylinder and an equivalent strip foundation: (a)-(d) vertical load; (b)-(e) 20° inclined load measured from the vertical direction; (c)-(f) 30° inclined load measured from the vertical direction

Figure 6 presents the failure mechanisms obtained from upper bound analysis for the cylinder with z/D = 0.5 along with the mechanism for the equivalent strip foundation. When the external load is along the vertical direction (Figure 6(a) and 6(d)), the supporting soils beneath the cylinder and strip foundation both exhibit deformation pattern similar to general failure mode for a shallow foundation (Terzaghi, 1943; Vesic, 1973). Nevertheless, the failure zone under a cylinder extends to greater depth and to a larger horizontal distance than that for strip foundation, thereby mobilizing greater vertical resistance. As the loading direction deviates from the vertical direction (i.e., Figures 6(b) and 6(e), and Figures 6(c) and 6(f)), although the failure patterns for the cylinder and strip foundation are similar, the loaded cylinder activates substantially greater passive resistance horizontally than the corresponding strip foundation, due to the influence of embedment and geometry. This difference explains why a relatively deeply embedded cylindrical object can develop larger resistance against horizontal loading than its equivalent shallow foundation.

Figure 7 shows the same V-H failure envelopes presented in Figure 6 but with V and H both normalized by the ultimate vertical bearing capacity $V = V_{max}$ corresponding to H = 0. It can be seen that the normalized failure envelopes predicted by the equivalent foundation model reasonably approximates those for an actual cylinder, even when the embedment depth is relatively large. This observation implies that the shallow foundation analogy can be used with some level of confidence in engineering practice to estimate the limit loads acting on cylinder objects, provided the maximum vertical capacity resistances V_{max} is available, or at least can be satisfactorily approximated.

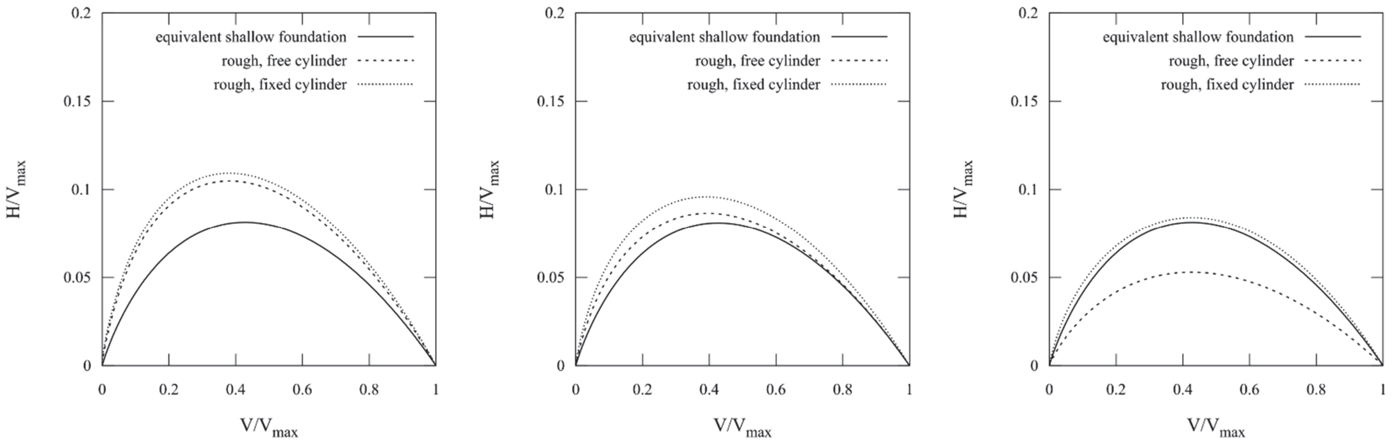

Fig. 7. Comparison of normalized limit loads acting on cylinder with those for an equivalent strip foundation: (a) z/D = 0.5; (b) z/D = 0.3; (c) z/D = 0.1

5. INFLUENCE OF CYLINDER ROTATION ON LIMIT LOAD

From Figure 5(c) and Figure 7(c), it can be seen that a cylinder that is constrained against rotation can mobilize higher resistance compared with a cylinder that can rotate freely. This gain of resistance is even more pronounced when the embedment depth is small. This observation is of practical importance, because it implies that the release of the rotational constraint for a loaded cylinder deteriorates its capacity against external loading, or reduces the resistance. Moreover, for the design of off-road vehicles, the above finding implies that a wheel's relative rotation rate can influence the contact forces, and thus also the thrust available, or resistance, depending on whether the wheel is towed or driven (cf. Hambleton and Stanier, 2017).

Figure 8(a) illustrates the planar force equilibrium of a partially embedded cylinder with constrained rotation. Due to the rotation constraint, a torque M is developed at the center of the cylinder to maintain moment equilibrium, and a contact moment M_b can also develop on the soil-cylinder interface.

A key conclusion of this study is that, for a cylinder with shallow embedment that is constrained against rotation, the contact moment M_b is approximately zero. This is deduced from the high level of agreement with the solution obtained using the shallow foundation analogy, for which it was assumed that there is no moment applied to the equivalent shallow foundation.

On the other hand, if the cylinder can rotate freely (Figure 8(b)), no moment M is applied at the center of the cylinder. As a consequence, to maintain equilibrium of the cylinder, a contact moment M_b must be mobilized at the interface between the cylinder and soil, and the value of M_b can be evaluated through:

$$M_b = H\left(\frac{D}{2} - z\right) \tag{5}$$

Previous studies on the bearing capacity of shallow foundations subjected to general loading (Gottardi and Butterfield, 1993; Taiebat and Carter, 2000; Gourvenec, 2007) have shown that the addition of a moment will diminish its capacity to resist horizontal and vertical forces. These findings are consistent with the behavior shown in Figures 5(c) and 7(c), in which the equivalent foundation model, which does not consider the moment, overestimates the limits load acting on cylinder. Moreover, Eq. (5) implies that the additional moment M_b exists only when the horizontal force H is not zero. This is also consistent with the behavior shown in Figures 5(c) and 7(c), where it is shown that rotational constraint has a negligible influence on the maximum vertical load. From the point view of practice, the above observations suggest that to accurately predict the limit loads on a rotating cylindrical object, a moment must be considered in the equivalent shallow foundation model in addition to vertical and horizontal forces.

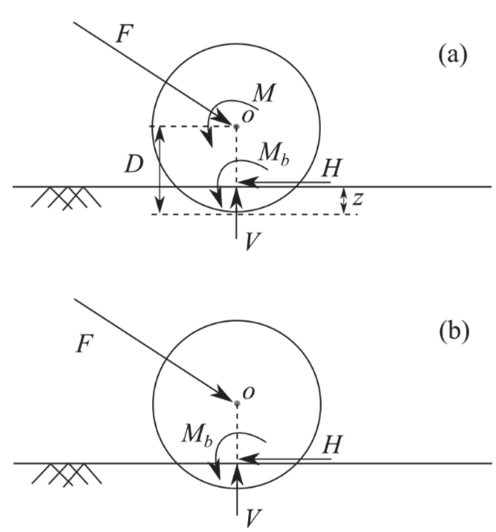

Fig. 8. Schematic showing the force equilibrium for (a) a cylinder with no rotation; (b) a cylinder with free rotation.

6. CONCLUDING REMARKS

The goal of this work is to examine whether an equivalent shallow foundation model, which is commonly utilized in practice, can provide satisfactory predictions regarding limit loads acting on partially embedded cylindrical objects subjected to combined vertical and horizontal loading. For this purpose, finite element limit analysis has been used to investigate the limit loads acting on (a) a cylinder whose geometry has been modeled precisely and (b) an equivalent shallow foundation.

The main conclusions that can be draw from this work are summarized as follows:

1) The limit loads that a partially embedded cylinder can sustain (or resistances mobilized with relative motion) depend on the embedment depth, the roughness at soil-structure interface, and the rotation of the cylinder. As embedment depth of the cylinder increases, the capacity also increases. When embedded at the same depth, a cylinder with a rough interface can sustain significantly larger vertical and horizontal loads than that with a smooth interface.

2) The equivalent shallow foundation model can reasonably approximate the limit loads for a cylinder subjected to combined vertical and horizontal loading, provided that the embedment depth is relatively small. The best match is observed for a cylinder that has very shallow embedment and is prohibited from rotation. For a relatively deeply embedded cylinder ($z/D > 0.3$), simplifying the cylinder as a strip foundation significantly underestimates the limit load. Nevertheless, the simplified model can reasonably approximate the failure envelope for a deeply embedded cylinder when it is normalized by the ultimate vertical capacity $V_{\max}$, which must then be known in order to complete any calculations.

3) Under combined vertical and horizontal loading, soils beneath a cylinder and an equivalent shallow foundation exhibit similar failure mechanisms. When the cylinder is deeply embedded, the failure mechanism extends farther into the soil, and a greater passive failure zone can be mobilized compared to the corresponding equivalent shallow foundation, resulting in greater lateral capacity.

4) A cylinder that is constrained against rotation can mobilize noticeably higher resistance compared with a cylinder that can rotate freely, especially when the embedment depth is small. This degradation of bearing capacity (reduction in resistance) due to the cylinder's rotation can be attributed to a contact moment acting along the interface between the cylinder and the soil.

A potential future study extending the work presented in this paper could focus on developing an equivalent foundation model in which the contact moment is considered, in addition to vertical and horizontal force. Such a generalized model would be useful not only in the analysis of pipelines but also in the area of off-road vehicle mobility.

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